General Incomplete Time Series Analysis via Patch Dropping Without Imputation

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Abstract

Missing values in multivariate time series data present significant challenges to effective analysis. Existing methods for multivariate time series analysis either ignore missing data, sacrificing performance, or follow the impute-then-analyze paradigm, which suffers from redundant training and error accumulation, leading to biased results and suboptimal performance. In this paper, we propose INTER, a novel end-to-end framework for incomplete multivariate time series analysis, which bypasses imputation by leveraging pre-trained language models to learn the distribution of incomplete time series data. INTER incorporates two novel components: the *missing-rate-aware time series patch-dropping* (MPD) strategy and the *missing-aware Transformer block*, both of which we propose to enhance model generalization, robustness, and the ability to capture underlying patterns in the observed incomplete time series. Moreover, we theoretically prove that the MPD strategy exhibits lower sample variance for time series with the same dropout rate compared to other dropping strategies. Extensive experiments on 11 public real-world time series datasets demonstrate that INTER improves accuracy by over 20% compared to state-of-the-art methods, while maintaining *competitive* computational efficiency.

1 Introduction

Multivariate time series analysis [Wen *et al.*, 2022] is widely used in extensive real-world applications, e.g., physiologic signals analysis [Moody *et al.*, 2011], stock price forecasting [Wang *et al.*, 2022], and anomaly detection [Franceschi *et al.*, 2019]. However, real-world multivariate time series data are usually imperfect and incomplete due to failures in data collection devices [Miao *et al.*, 2021; Miao *et al.*, 2022] and an unstable system environment [Wu *et al.*, 2022; Wu *et al.*, 2023]. For instance, the publicly available medical time series dataset *PhysioNet* [Silva *et al.*, 2012] has an average missing rate exceeding 80%. This incompleteness poses considerable difficulties for time series analysis, including (1) missing values that hinder the functionality of certain analytical methods, (2) disrupted temporal dependencies that lead to model bias, and (3) reduced model robustness that causes overfitting to noise or incomplete data, ultimately resulting in unstable performance in real-world applications. Given these difficulties, it is crucial to propose an effective solution for the *Incomplete Multivariate Time Series Analysis* (IMTSA) task, which improves the performance of time series analysis in the presence of missing data.

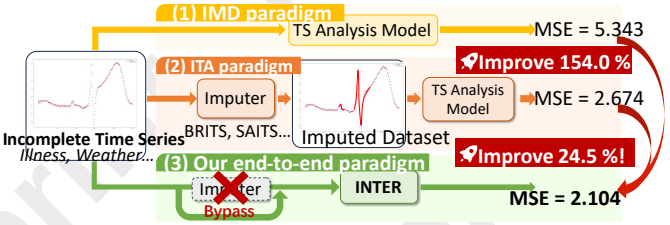


Figure 1: Traditional paradigm vs ours.

Existing Multivariate Time Series analysis methods are primarily designed for the *full-knowledge* scenario, which assumes access to *completely observed* datasets. For the IMTSA task, current solutions follow two flawed paradigms: (1) *Ignoring missing data (IMD)*: Performing downstream analysis without addressing missing values, which leads to severely degraded performance. (2) *Impute-then-analyze (ITSA)*: Imputing missing data before analysis, which suffers from redundant training and error accumulation, resulting in biased results and suboptimal performance.

Motivating Example. To better illustrate the challenges of incomplete multivariate time series analysis, we consider the widely used public medical dataset *PhysioNet*, which contains ICU patient records for tasks such as mortality prediction and clinical outcome analysis. Figure 1 compares the performance of existing paradigms for time series analysis on this dataset. The IMD paradigm that directly performs downstream analysis without handling missing values results in severely degraded performance. The ITSA paradigm first applies a standard imputation method (e.g., MICE [Buuren and Groothuis-Oudshoorn, 2010]) to fill in missing values, followed by a state-of-the-art model, such as a Transformer-based classifier [Lim and Zohren, 2021]. While this paradigm offers slight improvements, it suffers from error propagation introduced during the imputation phase, leading to biased

analysis and suboptimal results.

To address the limitations of the above two paradigms, we propose a novel end-to-end framework, named INTER, which introduces a new paradigm for incomplete multivariate time series analysis by bypassing the imputation process. As shown in Figure 1(c), INTER directly analyzes incomplete time series data without requiring separate imputation. By eliminating error accumulation and effectively capturing missing patterns, INTER achieves significantly better accuracy and robustness results than existing paradigms.

However, implementing INTER faces two challenging problems. First, *how can we effectively extract missing patterns and recover critical information from missing parts without performing explicit imputation?* (**CH1**) The challenge lies in the fact that missing data patterns often exhibit complex temporal and spatial dependencies, making them difficult to model directly. Moreover, skipping the imputation step can render traditional feature extraction methods ineffective. Therefore, it is essential to design a mechanism capable of capturing both explicit and implicit missing patterns, which will enable accurate representation of incomplete time series. Second, *how can we effectively adapt and transfer the knowledge derived from the missing data to diverse multivariate time series analysis tasks?* (**CH2**) Incomplete time series occur across various downstream tasks, such as forecasting, anomaly detection, and imputation. When dealing with datasets with high missing rates, excessive noise in such datasets can prevent downstream time series analysis models from even bootstrapping effectively. Furthermore, different tasks require different utilizations of the missing information, and a lack of alignment mechanisms makes it difficult to adapt and fine-tune the missing information for each specific downstream task

To address these two challenges, INTER consists of two key modules: the *missing-aware patch learning* (MPL) module and the *incomplete time series analysis* (ITSA) module, each designed to tackle one of the challenges independently. In the MPL module, we devise a novel *missing-rate-aware patch dropping* (MPD) strategy and a *missing-aware Transformer block*, both of which we propose to enhance model generalization, robustness, and the ability to capture underlying patterns in the observed incomplete time series. In the ITSA module, we fine-tune a pre-trained language model (PLM) with a novel element-wise loss function. By leveraging the pre-trained knowledge from the PLM, ITSA captures the underlying structure of incomplete time series data and refines it using the element-wise loss function, which is designed to minimize the interference from noise in the missing data. This process enables INTER to effectively learn both the distributions of observed data and the missing state distributions from incomplete time series.

Our main contributions are as follows: (1) *Paradigm:* We propose INTER, an end-to-end, missing-aware framework for the IMTSA problem that bypasses imputation and achieves exceptional performance. To the best of our knowledge, this is the first attempt to develop an end-to-end framework for incomplete multivariate time series analysis. (2) *Model:* In INTER, we devise two novel modules, i.e., MPT based on missing-aware transformer and ITSA based on pre-train language model, to address the two challenges, respectively. (3) *Theory:* We propose a novel *missing-state-aware patch-dropping* (MPD) strategy and theoretically prove that MPD achieves lower sample variance for time series with the same dropout rate compared to other dropping strategies. (4) *Experiment:* We conduct extensive experiments on 11 real-world datasets, demonstrating that INTER outperforms state-of-the-art methods in terms of accuracy while maintaining comparable computational efficiency.

2 Related Work

Existing *time series imputation* models serve as the foundation for most multivariate time series *analysis* methods. They include statistical ones, attention-based ones, deep generative model based ones, and neural ordinary differential equation (NODE) based ones. In particular, the statistical time series imputation algorithms substitute missing values with the statistics, e.g., zero, mean, and last observed value or simple statistical models, including ARIMA [Bartholomew, 1971]. The attention mechanism based time series imputation methods contain TransI [Vaswani *et al.*, 2017] and SAITS [Du *et al.*, 2023]. The deep generative model based time series imputation methods use autoencoder and generative adversarial network to impute missing values, including BRITS [Cao *et al.*, 2018] and CSDI [Tashiro *et al.*, 2021]. The NODE-based methods model the dynamics over time by virtue of the continuous nature of NODE, such as SaShiMi [Goel *et al.*, 2022] and LS4 [Zhou *et al.*, 2023a].

Multivariate time series analysis methods have attracted extensive research focus from the AI community [Liang *et al.*, 2024], and these approaches can be broadly categorized into three types. With the development of technologies such as Transformers, pretrained language models, and mixture of experts[Vaswani *et al.*, 2017; Feng *et al.*, 2024; Feng *et al.*, 2025], Transformer-based models have become increasingly effective in time series analysis, including Informer [Zhou *et al.*, 2021], PatchTST [Nie *et al.*, 2022], OneFitsAll [Zhou *et al.*, 2023b] and Timer[Liu *et al.*, 2024]. The second category consists of non-Transformer-based models, such as CNN-based approaches like TimesNet [Wu *et al.*,] and TriD-MAE [Zhang *et al.*, 2023], which capture spatial and temporal patterns using convolutions. With the development of GNN methods [Zhu *et al.*, 2023; Liao *et al.*, 2024], models like FourierGNN [Yi *et al.*, 2023a] have started leveraging graph structures for multivariate time series forecasting. MLP-based models like FreTS [Yi *et al.*, 2023b] and TSMixer [Ekambaram *et al.*, 2023] focus on learning complex feature interactions in time series data. FilterNet [Yi *et al.*, 2024] employs frequency filtering techniques to enhance forecasting performance. The third category involves diffusion-based models, such as D^3VAE [Li *et al.*, 2022] and DiffTime [Coletta *et al.*, 2024], which model time series dynamics by progressively adding and removing noise, making them effective for handling missing or uncertain data.

3 Problem Definition

The input multivariate time series dataset contains a set of samples $\mathcal{D} = \{\mathbf{X}_1, \cdots, \mathbf{X}_N\}$ with d dimensions

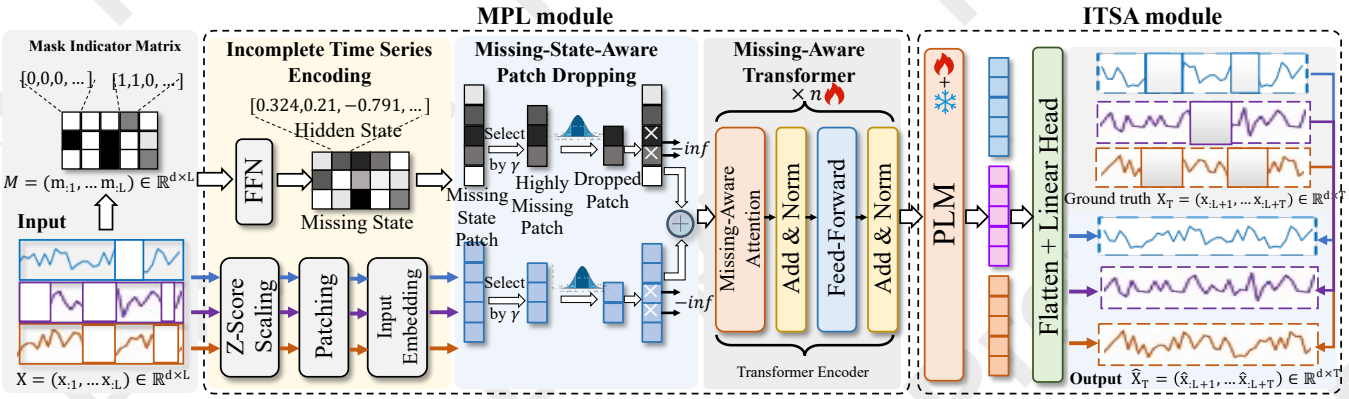


Figure 2: The architecture of INTER

and L timestamps. Formally, $\mathbf{X} = (\mathbf{x}_1, \cdots, \mathbf{x}_d)^\top = (\mathbf{x}_{:1}, \cdots, \mathbf{x}_{:L}) \in \mathbb{R}^{d \times L}$ with $\mathbf{x}_i$ being $(x_{i1}, \cdots, x_{iL})$. In particular, x_{ij} is the i-th feature value of $\mathbf{X}$ at the j-th timestamp, which is probably missing in the incomplete multivariate time series dataset.

Definition 1. Incomplete multivariate time series, IMTS. *To encode the missing information of each sample $\mathbf{X}$ in $\mathcal{D}$, a mask matrix $\mathbf{M} = (\mathbf{m}_1, \cdots, \mathbf{m}_d)^\top = (\mathbf{m}_{:1}, \cdots, \mathbf{m}_{:L}) \in \{0,1\}^{d \times L}$ w.r.t. $\mathbf{X}$ is used to indicate whether the values in $\mathbf{X}$ exist or not. $\mathbf{m}_i = (m_{i1}, \cdots, m_{iL})$, and m_{ij} being 0 or 1 means that, x_{ij} is missing or observed.*

Definition 2. Incomplete multivariate time series representation learning, IMTSRL. *Given an incomplete multivariate time series $\mathbf{X}$ with its mask matrix $\mathbf{M}$, the goal of incomplete time series representation learning is to train a neural network model (i.e., encoder) $f : \mathbb{R}^d \mapsto \mathbb{R}^{d_r}$, where d_r is the dimensionality of the representation, such that the representation $\mathbf{z} = f(\mathbf{X})$ can be informative for downstream analysis tasks, e.g., multivariate time series classification.*

Based on probability theory [Feng *et al.*, 2023], we claim that, during model optimization, the analysis approach leveraging IMTSRL proves more effective than the imputation-then-analysis solution, as highlighted in Observation 1.

Framing the classification task as the foundation for IMTSRL, the objective is to train a classifier $\mathcal{H}$ that uses the IMTS data $\mathbf{X} \in \mathcal{D}$ to predict the label y. For ease of representation, we restrict our domain to a three-class task in this part, i.e., $y \in \{0, 1, 2\}$. Inspired by [Feng *et al.*, 2023], we evaluate the classifier's performance differences across various categories in the discrete attribute $\mathbf{K}$ in $\mathbf{X}$ that exhibits a complex missing mechanism, e.g., MAR and MNAR. Such a classifier is to solve a constrained optimization problem $\max_{\mathcal{H}} \mathbb{E}[\mathbb{I}(\mathcal{H}(\mathbf{X}) = \mathbf{Y})]$ subject to $\mathrm{Disc}(\mathcal{H}) \leq \epsilon$, where $\epsilon \geq 0$ is a tolerance threshold. When the optimization is over binary mappings, the optimal solution of the constrained optimization problem only depends on the data distribution $P_{\mathbf{K},\mathbf{X},\mathbf{Y}}$ and ϵ. We denote the optimal solution of this problem by $F_\epsilon(P_{\mathbf{K},\mathbf{X},\mathbf{Y}})$. Here, $\mathbb{I}$ is the indicator function, returning 1 if $\mathcal{H}$ correctly predicts $\mathcal{H}(\mathbf{X}) = \mathbf{Y}$, and 0 otherwise. $\mathrm{Disc}(\mathcal{H})$ represents the model's disparity in performance across different categories in $\mathbf{K}$, i.e., $\mathrm{Disc}(\mathcal{H}) = \max_{y,\hat{y},k,k'} |\Pr(\mathcal{H}(\mathbf{X}) = \hat{y} \mid \mathbf{Y} = y, \mathbf{K} = k) - \Pr(\mathcal{H}(\mathbf{X}) = \hat{y} \mid \mathbf{Y} = y, \mathbf{K} = k')|$. Moreover, we use mutual information [Kraskov *et al.*, 2004] to quantify the dependence between the missing matrix $\mathbf{M}$ and the label $\mathbf{Y}$, i.e., $I(\mathbf{M}; \mathbf{Y}) \triangleq \sum_{\mathbf{m} \in \mathbf{M}} \sum_y P_{\mathbf{M},\mathbf{Y}}(\mathbf{m}, y) \log P_{\mathbf{M},\mathbf{Y}}(\mathbf{m}, y) / (P_{\mathbf{M}}(\mathbf{m}) P_{\mathbf{Y}}(y))$. The detailed proof can be found in Appendix A.

Observation 1. *Suppose that the incomplete IMTS dataset $\mathbf{X}$ with the mask matrix $\mathbf{M}$ and label set $\mathbf{Y}$ consists of a single discrete attribute $\mathbf{K}$, which is subject to a complex missingness mechanism, such as MAR or MNAR. Let α represent the probability that the outcome of the three-class classification is not equal to 0 for a given sample. For any ϵ and $\alpha \in (\frac{1}{2}, 1)$, there exists a data distribution $P_{\mathbf{K},\mathbf{X},\mathbf{Y}}$ such that the optimal solution $F_\epsilon(P_{\mathbf{K},\hat{\mathbf{X}},\mathbf{Y}})$ is less than or equal to the optimal solution $F_\epsilon(P_{\mathbf{K},\mathbf{X},\mathbf{Y}})$, i.e., $\sup_{f_{imp}} F_\epsilon\left(P_{\mathbf{K},\hat{\mathbf{X}},\mathbf{Y}}\right) \leq F_\epsilon\left(P_{\mathbf{K},\mathbf{X},\mathbf{Y}}\right) - \alpha$, where f_{imp} is the mapping function of the IMTS imputation model. The imputed data $\hat{\mathbf{X}} = f_{imp}(\mathbf{X})$.*

4 Methodology

4.1 Overall

Our proposed INTER framework consists of two key modules: *Missing-aware Patch Learning* (MPL) and *Incomplete Time Series Analysis* (ITSA). The architecture of INTER is shown in Figure 2. Specifically, INTER takes an incomplete multivariate time series $\mathbf{X} = \{x_1, \ldots, x_L\} \in \mathbb{R}^{d \times L}$ as input and processes it sequentially through the MPL and ITSA to generate outputs for downstream tasks. Figure 2 illustrates an example of the output for a time series forecasting task, $\hat{\mathbf{X}}^T = \{x_{L+1}, \ldots, x_{L+T}\}$. First, the MPL module includes three components: *incomplete time series encoding* (TSE), *missing-state-aware patch dropping* (MPD), and *missing-aware Transformer* (MAT). TSE applies mainstream time series processing components for standard preprocessing and, moreover, learns the missing patterns in each incomplete multivariate time series. MPD adopts a novel patch-dropping strategy to reduce noise, enhancing model generalization and robustness. MAT learns the observed data distribution and the missing state distribution based on a

transformer-based encoder. Second, the ITSA module leverages pre-trained language models to provide prior knowledge for bootstrapping the learning of underlying observed time series patterns. It then uses a flattening layer and linear head to project the learned representations onto specific downstream tasks. Additionally, an element-wise loss function is introduced to mitigate the risk of learning incorrect information from missing values. The pseudo-code of INTER can be found in Appendix B.

4.2 MPL: Missing-aware Patch Learning

Traditional multivariate time series analysis models typically rely on imputation algorithms to handle missing data when performing IMTSA tasks. However, this paradigm introduces unnecessary training redundancy, error propagation, and ultimately leads to suboptimal performance. To address these issues, we propose the MPL module, which operates directly on incomplete multivariate time series, extracting patches containing missing values and observed time series data. This process enables the model to learn the underlying patterns of missing data and generate high-level hidden representations *without imputation*. Next, we elaborate on the three key components of the MPL module.

TSE: Incomplete Time Series Encoding. The TSE serves two primary purposes for processing incomplete time series data: (1) extracting missing state information and (2) generating embedded representations of the time series.

For the first purpose, TSE employs a *feedforward neural network* (FFN) to extract the missing-state matrix $\mathbf{M}^s \in \mathbb{R}^{d\times L}$ from the mask indicator matrix $\mathbf{M} \in \{0,1\}^{d\times L}$. To achieve this, the mask indicator matrix is divided into patches P, which are then passed through the FFN to learn the missing state matrix, where for the i-th patch, the missing state $\mathbf{m}_i^{s,p} = f_{\text{FFN}}(\mathbf{m}_i^p)$. This learned missing state is then used in the MPD module to provide a missing pattern of each patch for further refinement.

For the second purpose, TSE first normalizes the incomplete time series using Z-score scaling. Specifically, for each time series $\mathbf{x}_i \in \mathbf{X}$ with its corresponding mask vector $\mathbf{m}_i \in \mathbf{M}$, we calculate the mean μ_i and standard deviation σ_i across the observed elements, i.e.,

$$\mu_i = \frac{\sum_{j=1}^{L} x_{i,j}\cdot m_{i,j}}{\sum_{j=1}^{L} m_{i,j}}, \quad \sigma^{i2} = \frac{\sum_{j=1}^{L}(x_{i,j}-\mu_i)^2\cdot m_{i,j}}{\sum_{j=1}^{L} m_{i,j}}.$$

Then, for each observed value in $\mathbf{x}_i$, we subtract μ_i and divide by σ_i. This normalization reduces variability, promoting stable learning across the data. Note that a de-normalization procedure is applied to the forecasted data to ensure that the predicted time series are interpretable in relation to the historical data. After normalization, we divide the time series into non-overlapping patches $\ddot{\mathbf{x}}_i^p \in \mathbb{R}^{P\times N}$, where each patch corresponds to a subseries-level segment of the time series [Nie *et al.*, 2022]. Finally, we apply an Input Embedding layer to project the time series embedding $\ddot{\mathbf{x}}_i^p$ to $\mathbf{x}_i^p$, where $\mathbf{x}_i^p$ is mapped to the required dimensions for the subsequent pre-trained model [Zhou *et al.*, 2023b]. This embedding captures complex dependencies across time steps and is crucial for facilitating the learning process in later modules.

MPD: Missing-state-aware patch dropping. The core idea of the MPD is to perform dropping based on the missing states of the time series data. Unlike traditional dropout methods, which apply a uniform dropping probability across all data points, MPD introduces a *random incomplete patch dropping* strategy. This strategy directly applies dropping to incomplete time series patches, rather than individual time series values. Specifically, MPD drops incomplete time series patches at a rate of δ, such that, on average, $\delta \cdot P$ patches are masked in each time series. Moreover, the dropping probability δ_i^p for each patch is dynamically determined based on its missing rate γ_i^p, with patches that have a higher proportion of missing data being more likely to be dropped due to the significant noise they carry.

The dropping operation in MPD can be viewed as a patch-sampling process. For each patch $\mathbf{x}_i^p$ in the time series, an independent mask α_i is generated to determine whether the patch will be preserved, following a Bernoulli distribution:

$$\alpha_i \sim \text{Bernoulli}(1-\delta^p), \tag{1}$$

where δ^p is the dropping probability dynamically assigned to each patch $\mathbf{x}_i^p$, based on its missing rate γ^p. Specifically, the dropping probability δ^p for each patch is designed to be positively correlated with its missing rate γ^p, such that:

$$\delta^p = f(\gamma^p), \quad f(\gamma^p) \propto \gamma^p, \tag{2}$$

where $f(\gamma^p)$ is a function that increases with the patch's missing rate, enabling an adaptive dropping strategy that effectively addresses the challenges posed by patches containing a high proportion of missing values. Subsequently, for each patch $\mathbf{x}_i^p$ and its corresponding missing state $\mathbf{m}_i^{s,p}$, the mask α_i is generated based on the Bernoulli distribution with the dropping probability δ^p. Finally, the perturbed time series matrix is obtained by performing element-wise multiplication of each patch with its corresponding mask α_i.

The effectiveness and theoretical advantages of the MPD strategy are demonstrated through rigorous analysis. Specifically, Theorem 1 shows that the MPD strategy based on missing rates results in lower sample variance at the same dropout rate, contributing to a more stable training process. Therefore, the MPD strategy can serve as an additional regularization term of INTER, thereby improving the model's robustness. The detailed proof can be found in Appendix C.

Theorem 1. *Compared to two existing dropping strategies, i,e, (i) a value-based dropping strategy, and (ii) a random patch-dropping strategy based on uniform distribution, the MPD strategy achieves a smaller sample variance on time series with the same dropout rate.*

MAT: Missing-aware Transformer. The MAT module leverages an n-layer Transformer to process missing state-aware patches and time series patches, capturing hidden temporal patterns and producing a sequence of hidden representations $\{\mathbf{z}_i\}_{i=1}^N$ that integrate both observed data and missing state information.

First, the time series vector $\mathbf{x}_i^p$ and the corresponding missing state $\mathbf{m}_i^{s,p}$ are concatenated to form a combined representation for each patch, i.e., $\mathbf{z}_i^p = \text{concat}(\mathbf{x}_i^p, \mathbf{m}_i^{s,p})$. This embeds the missing state distribution into the time series

patches, enriching the input representation with information about both observed data and missing patterns.

Next, during the self-attention computation, patches marked as *dropped* need to be excluded by masking their attention scores. Therefore, we devise a missing-aware attention with a new score-updating mechanism. Specifically, given the attention score matrix $\mathbf{A} \in \mathbb{R}^{N \times N}$, the scores are updated as follows:

$$\mathbf{A}_{i,j} = \begin{cases} -\infty & \text{if } i \in \mathcal{D} \text{ or } j \in \mathcal{D}, \\ \mathbf{A}_{i,j} & \text{otherwise}, \end{cases} \tag{3}$$

where $\mathcal{D}$ is the dropped data. This ensures that dropped patches do not influence the attention mechanism, focusing the model only on valid patches.

Finally, the remaining patches are passed through the Transformer, which performs missing-aware self-attention and feedforward operations to generate the hidden representation for each patch, i.e., $\tilde{\mathbf{z}}_i^p = \text{Transformer}(\{\mathbf{z}_j^p \mid j \notin \mathcal{D}\})$. This enables the Transformer to aggregate temporal dependencies across patches while incorporating missing state information into the learned representations. The MAT module outputs the sequence of hidden representations $\tilde{\mathbf{Z}}^p = \{\tilde{\mathbf{z}}_i^p\}_{i=1}^{P-|\mathcal{D}|}$, where each vector effectively encodes both observed time series data and missing state information, providing a robust foundation for downstream tasks.

4.3 ITSA: Incomplete Time Series Analysis

After the MPL module, INTER employs the *incomplete time series analysis* (ITSA) module, which processes each patch independently in a sequence-to-sequence framework. ITSA leverages pre-trained parameters from NLP models, specifically GPT-2 [Radford *et al.*, 2019], to enhance the representation of incomplete time series data.

Architecture. In ITSA, time series patches $\tilde{\mathbf{Z}}^p$ from MPL are treated as sequence tokens and passed through the pre-trained language model (PLM). ITSA retains the PLM's positional embeddings and self-attention blocks. As most knowledge resides in the self-attention and FFN layers, these are frozen during fine-tuning, allowing only final layers to adapt and better capture temporal dependencies in time series data.

Specifically, the input to ITSA is a sequence of patches $\tilde{\mathbf{Z}}^p$. The PLM processes this as: $\mathbf{H} = \text{PLM}(\tilde{\mathbf{Z}}^p)$, where $\mathbf{H} \in \mathbb{R}^{N \times D}$ is the output hidden representation and D is the PLM hidden size. GPT-2 is used as the PLM in our experiments. During fine-tuning, only the output layers are updated, while self-attention and FFN remain frozen. The output $\mathbf{H}$ captures complex temporal patterns from incomplete multivariate time series. ITSA is designed to leverage PLM knowledge to model incomplete structures. Fine-tuning on incomplete patches improves the model's ability to learn latent temporal dependencies, helping INTER better handle missing values and uncover hidden patterns for downstream tasks.

Objective. On top of PLM, ITSA applies a flatten layer (FL) and a linear head (LH) to infer future time series: $\hat{\mathbf{x}}_i^T = (\hat{x}_i^1, \cdots, \hat{x}_i^d) = FL(LH(\mathbf{H}))$. where $\mathbf{H}$ is the ITSA output. Moreover, the ITSA module incorporates distinct objective functions specifically designed for various tasks, including forecasting (both long-term and short-term), anomaly detection, and imputation on incomplete time series data. Taking the incomplete time series forecasting task as an example, the ITSA module is designed to effectively learn the mapping between incomplete historical time series and future observations. To achieve this, we propose an incomplete time series Mean Squared Error (MSE) loss function, defined as:

$$\mathcal{L}_{\mathbf{MSE}} = \frac{1}{N} \sum_{\mathbf{X} \in \mathcal{X}} \left[\frac{\sum_{i=1}^{d} || \left(\mathbf{m}_i^T\right) \odot (\mathbf{x}_i^T - \hat{\mathbf{x}}_i^T)||_2^2}{||\mathbf{M}^T||_2^2} \right], \tag{4}$$

where $\odot$ is the element-wise multiplication. $||\cdot||_2^2$ represents the squared Euclidean norm, which is the sum of the squares of the elements of a vector. $\mathbf{x}_i^T$ is the ground-truth future data of the time series $\mathbf{x}_i \in \mathbf{X}$. $\mathbf{m}_i^T$ in $\mathbf{M}^T$ is the mask vector w.r.t. $\mathbf{x}_i^T$. The incomplete time series forecasting loss function $\mathcal{L}_{\text{MSE}}$ is designed to enforce the consistency between the true underlying data distribution in incomplete future time series and output data distribution. It can be easily generalized to other time series analysis tasks, e.g., the incomplete time series classification, as described in Appendix D.

5 Experiment

In this section, we evaluate the performance of our proposed model INTER on five tasks—long-term forecasting, short-term forecasting, imputation, classification, and anomaly detection—using 11 (in)complete multivariate time series datasets. The performance is compared with a total of 8 state-of-the-art time series analysis methods. All approaches were implemented in Python. The experiments were conducted on a server with an Intel Core 2.80GHz processor, 3 NVIDIA A40 GPUs, and 192GB RAM, running Ubuntu 18.04.

5.1 Experiment Settings

Metrics. We evaluate the effectiveness of models in time series forecasting tasks using the *mean square error* (MSE) and *mean absolute error* (MAE), where smaller metric values indicate better prediction performance. For time series classification tasks, we use *Accuracy* as the evaluation metric to measure model effectiveness. To evaluate the effectiveness of forecasting models on incomplete multivariate time series datasets, we randomly remove 50% of the observed values prior to model training. Each metric value is obtained by averaging the results of five experimental runs on each dataset.

Baselines. In the experiments, the baselines include six state-of-the-art time series forecasting methods, including: Informer [Zhou *et al.*, 2021], PatchTST [Nie *et al.*, 2022], TimesNet [Wu *et al.*,], OneFitsAll [Zhou *et al.*, 2023b], Timer [Liu *et al.*, 2024], and TriD-MAE [Zhang *et al.*, 2023]. Since almost all the above baselines cannot be trained with incomplete time series data, we employ five state-of-the-art multivariate time series imputation methods, i.e., Zero [Che *et al.*, 2018], BRITS [Cao *et al.*, 2018], TransI [Vaswani *et al.*, 2017], SAITS [Du *et al.*, 2023], and CSDI [Tashiro *et al.*, 2021] to impute missing values.

5.2 Comparison Study

Overall, the proposed method, INTER, outperforms other models across various tasks, including incomplete time series

Models		*Electricity*		*Weather*		*Exchange*		*Illness*	
		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Informer	Zero	0.352 ± 0.012	0.428 ± 0.009	0.463 ± 0.001	0.458 ± 0.012	0.524 ± 0.007	0.564 ± 0.003	5.706 ± 0.123	1.480 ± 0.051
	BRITS	0.358 ± 0.006	0.434 ± 0.004	0.514 ± 0.007	0.480 ± 0.005	0.522 ± 0.005	0.576 ± 0.007	5.552 ± 0.456	1.591 ± 0.032
	TransI	0.356 ± 0.015	0.432 ± 0.015	0.462 ± 0.014	0.444 ± 0.009	0.504 ± 0.009	0.570 ± 0.005	5.168 ± 0.789	1.510 ± 0.076
	SAITS	0.352 ± 0.01	0.430 ± 0.007	0.510 ± 0.019	0.475 ± 0.016	0.549 ± 0.001	0.588 ± 0.009	5.706 ± 0.012	1.616 ± 0.091
	CSDI	\	\	\	\	0.508 ± 0.006	0.570 ± 0.001	5.278 ± 0.345	1.532 ± 0.043
PatchTST	Zero	0.222 ± 0.018	0.320 ± 0.018	0.201 ± 0.015	0.279 ± 0.007	0.582 ± 0.008	0.580 ± 0.004	3.943 ± 0.678	1.402 ± 0.025
	BRITS	0.201 ± 0.02	0.299 ± 0.012	0.146 ± 0.01	0.190 ± 0.014	0.143 ± 0.003	0.288 ± 0.008	2.729 ± 0.901	1.104 ± 0.087
	TransI	0.207 ± 0.01	0.303 ± 0.016	0.163 ± 0.003	0.204 ± 0.018	0.362 ± 0.012	0.471 ± 0.006	3.024 ± 0.234	1.216 ± 0.036
	SAITS	0.198 ± 0.018	0.296 ± 0.008	0.143 ± 0.012	0.181 ± 0.01	0.266 ± 0.004	0.367 ± 0.003	2.520 ± 0.567	1.052 ± 0.019
	CSDI	\	\	\	\	0.05 ± 0.007	0.156 ± 0.002	8.191 ± 3.191	1.640 ± 0.057
TimesNet	Zero	0.202 ± 0.014	0.311 ± 0.014	0.473 ± 0.004	0.480 ± 0.015	0.978 ± 0.004	0.753 ± 0.01	5.253 ± 0.135	1.620 ± 0.049
	BRITS	0.175 ± 0.019	0.284 ± 0.019	0.144 ± 0.017	0.198 ± 0.017	0.194 ± 0.009	0.352 ± 0.007	3.949 ± 0.246	1.366 ± 0.067
	TransI	0.183 ± 0.02	0.289 ± 0.002	0.159 ± 0.002	0.213 ± 0.004	0.408 ± 0.011	0.508 ± 0.004	4.436 ± 0.369	1.499 ± 0.008
	SAITS	0.172 ± 0.012	0.282 ± 0.006	0.144 ± 0.009	0.196 ± 0.009	0.114 ± 0.005	0.263 ± 0.005	3.926 ± 0.789	1.354 ± 0.099
	CSDI	\	\	\	\	0.061 ± 0.008	0.180 ± 0.001	7.140 ± 0.012	1.805 ± 0.027
OneFisAll	Zero	0.267 ± 0.01	0.359 ± 0.011	0.277 ± 0.005	0.361 ± 0.001	0.720 ± 0.006	0.677 ± 0.009	5.009 ± 0.395	1.571 ± 0.041
	BRITS	0.213 ± 0.017	0.301 ± 0.014	0.159 ± 0.016	0.206 ± 0.008	0.133 ± 0.003	0.291 ± 0.008	3.579 ± 0.678	1.421 ± 0.056
	TransI	0.219 ± 0.012	0.305 ± 0.005	0.175 ± 0.007	0.217 ± 0.019	0.366 ± 0.007	0.487 ± 0.006	3.287 ± 0.901	1.308 ± 0.013
	SAITS	0.21 ± 0.02	0.296 ± 0.019	0.159 ± 0.011	0.198 ± 0.006	0.083 ± 0.01	0.216 ± 0.003	4.328 ± 0.234	1.531 ± 0.078
	CSDI	\	\	\	\	0.052 ± 0.001	0.161 ± 0.002	6.455 ± 0.567	1.638 ± 0.009
Timer	Zero	0.272 ± 0.016	0.378 ± 0.016	0.290 ± 0.019	0.411 ± 0.007	0.800 ± 0.004	0.737 ± 0.007	5.343 ± 0.89	1.720 ± 0.083
	BRITS	0.186 ± 0.013	0.281 ± 0.007	0.179 ± 0.003	0.236 ± 0.014	0.173 ± 0.008	0.311 ± 0.001	3.549 ± 0.135	1.461 ± 0.045
	TransI	0.193 ± 0.002	0.285 ± 0.003	0.215 ± 0.008	0.260 ± 0.018	0.386 ± 0.002	0.457 ± 0.005	3.337 ± 0.246	1.318 ± 0.015
	SAITS	0.167 ± 0.016	0.267 ± 0.015	0.184 ± 0.015	0.212 ± 0.005	0.059 ± 0.006	0.188 ± 0.01	2.674 ± 0.369	1.091 ± 0.06
	CSDI	\	\	\	\	0.053 ± 0.007	0.196 ± 0.012	4.288 ± 0.789	1.501 ± 0.031
TriD-MAE		0.311 ± 0.018	0.391 ± 0.004	0.233 ± 0.027	0.322 ± 0.091	1.388 ± 0.200	0.940 ± 0.009	8.290 ± 0.012	2.170 ± 0.092
INTER (Ours)		**0.147** ± 0.01	**0.250** ± 0.018	**0.139** ± 0.017	**0.192** ± 0.020	**0.049** ± 0.003	**0.158** ± 0.013	**2.104** ± 0.345	**0.984** ± 0.007

Table 1: Long-term forecasting performance comparison under different datasets

long-term and short-term forecasting, *classification*, *anomaly detection*, and *imputation*. Due to space limitations, the experimental results for *anomaly detection* and *imputation* tasks are presented in Appendix E.1. These results highlight the effectiveness and robustness of the proposed INTER framework in handling incomplete time series data.

In addition, we investigate the influence of different elements of INTER on the prediction performance over the incomplete multivariate time series data, i.e., **Ablation Study**. The detailed experimental results are described in Appendix E.2. We can observe that, each component of INTER positively affects performance. Then, we study the **Effect of Missing Rate** (i.e., how many features/values in multivariate time series data are dropped) on the time series analysis performance. The detailed experimental results are described in Appendix E.3. One can observe that, INTER basically achieves the best forecasting performance in each case.

Incomplete Long-term Time Series Forecasting

Setups. To fully evaluate model performance in forecasting, we adopt two benchmark types: long-term and short-term. For the long-term setting, we use four widely-used public multivariate time series datasets: *Electricity* [Gasparin *et al.*, 2022], *Weather* [Zhou *et al.*, 2021], *Exchange* [Zhang and Berardi, 2001], and *Illness* [Zhou *et al.*, 2021], covering four real-world scenarios.

Results. Table 1 presents the experimental results of incomplete time series forecasting methods. For *Electricity* and *Weather*, results of CSDI-based baselines are unavailable (denoted as "\") due to exceeding the 10^5-second time limit.

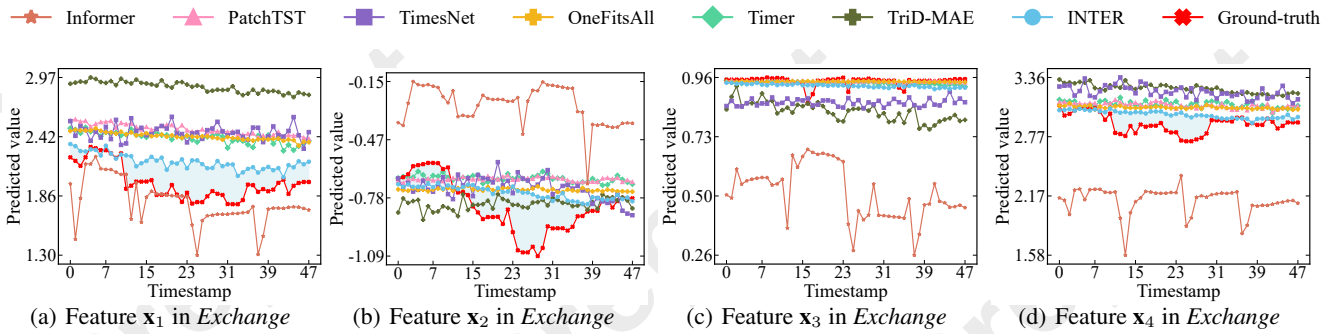
Models		*M4-Yearly*		*M4-Monthly*	
		SMAPE	MASE	SMAPE	MASE
TimesNet	Zero	18.523	7.512	17.421	2.980
	SAITS	15.460	5.079	14.371	1.922
OneFitsAll	Zero	17.531	7.015	17.283	4.965
	SAITS	15.419	5.033	14.320	1.870
Timer	Zero	18.586	7.180	17.491	5.046
	SAITS	15.510	5.124	14.478	1.982
INTER (Ours)	\	**15.322**	**4.913**	**14.106**	**1.790**

Table 2: Short-term forecasting performance

This is caused by the much higher complexity of CSDI's score-based diffusion model compared to other imputation methods.

It can be observed that INTER substantially outperforms all baselines. In forecasting accuracy (MSE and MAE), INTER surpasses the best imputation-based method (Timer with SAITS) by 20.91% on average, reaching 24.46% on *Illness* in terms of MSE. Compared to TriD-MAE, which integrates built-in imputation, INTER achieves an average improvement of 76.20% in MSE, demonstrating a significant accuracy gain. This is attributed to INTER's use of an effective analysis model that combines a missing-state-aware dropping strategy with an incomplete forecasting loss, thus boosting accuracy.

Visualization. Figure 3 visualizes forecasting results for the top 4 features of a randomly selected sample from *Exchange*. It clearly shows that INTER yields much better accuracy than



(a) Feature $\mathbf{x}_1$ in *Exchange* (b) Feature $\mathbf{x}_2$ in *Exchange* (c) Feature $\mathbf{x}_3$ in *Exchange* (d) Feature $\mathbf{x}_4$ in *Exchange*

Figure 3: Visualization of incomplete multivariate time series forecasting on *Exchange*

Models	Impu.	*Physionet*	*Heartbeat*	*JapaneseVowels*
Informer 2021	Zero	0.774	0.724	0.904
	BRITS	0.786	0.730	0.962
	SAITS	0.803	0.735	0.962
PatchTST 2022	Zero	0.867	0.780	0.910
	BRITS	0.877	0.737	0.965
	SAITS	0.875	0.780	0.964
Dlinear 2023	Zero	0.769	0.717	0.874
	BRITS	0.782	0.722	0.912
	SAITS	0.797	0.727	0.929
TimesNet 2023	Zero	0.865	0.722	0.906
	BRITS	0.878	0.751	0.964
	SAITS	0.874	0.756	0.963
OneFitsAll 2023	Zero	0.873	0.728	0.912
	BRITS	0.886	0.761	0.966
	SAITS	0.882	0.757	0.965
Timer 2024	Zero	0.872	0.734	0.876
	BRITS	0.885	0.767	0.914
	SAITS	0.881	0.763	0.931
TriD-MAE	\	0.776	0.723	0.856
INTER (Ours)	\	**0.897**	**0.820**	**0.989**

Table 3: Classification accuracy under different datasets

baselines, consistently being closest to the ground truth, further confirming its strong forecasting ability on incomplete multivariate time series.

Incomplete Short-term Time Series Forecasting

Setups. For the short-term setting, we adopt the M4 [Makridakis, 2018] dataset and its representative subsets, including yearly and monthly collected univariate marketing data. The M4 dataset contains 100,000 time series at varying frequencies. We adopt three baselines with strong performance on this task: TimesNet, OneFitsAll, and Timer. For each impute-then-analysis method, we use two representative imputation algorithms, Zero and SAITS, to fill in the missing data.

Results. Table 2 presents the experimental results of multivariate time series analysis methods for the short-term forecasting task. It can be observed that INTER clearly outperforms all baselines, exceeding the best-performing method (i.e., OneFitsAll with SAITS) by an average of 2.90%, demonstrating its strong capability in short-term forecasting tasks.

Incomplete Time Series Classification

Setups. We adopt sequence-level classification to verify the model's capacity for high-level representation learning. We use three public real-world time series datasets: two representative multivariate datasets from the *UEA Time Series Classification Archive* [Bagnall *et al.*, 2018] (i.e., *Heartbeat* and *JapaneseVowels*), and one public medical dataset (i.e., *Physionet*) [Goldberger *et al.*, 2000]. We pre-process the datasets following the descriptions in [Zerveas *et al.*, 2021], where subsets vary in sequence length.

Results. Table 3 presents a comparative analysis of classification performance across the three datasets. INTER achieves the best performance in all cases, surpassing the best baseline, OneFitsAll with BRITS, by an average of 3.56% in accuracy. Compared with the analysis method incorporating built-in imputation, TriD-MAE, INTER achieves an average improvement of 14.91% in accuracy. Moreover, INTER demonstrates the most stable classification accuracy, further validating its effectiveness.

6 Conclusion

In this paper, we propose INTER, a novel end-to-end framework for incomplete multivariate time series analysis that bypasses traditional imputation methods by leveraging PLM to directly learn the distribution of incomplete time series data. We further provide a theoretical analysis showing that the MPD strategy in INTER achieves lower sample variance for time series with the same dropout rate compared to alternative dropping strategies, demonstrating its statistical advantage. Extensive experiments conducted on 11 publicly available real-world time series datasets show that INTER greatly beats the state-of-the-art methods in effectiveness.

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